## Technical Comments

Brief discussion of previous investigations in the aerospace sciences and technical comments on papers published in the Journal of Guidance, Control, and Dynamics are presented in this special department. Entries must be restricted to a maximum of 1000 words, or the equivalent of one Journal page including formulas and figures. A discussion will be published as quickly as possible after receipt of the manuscript. Neither the AIAA nor its editors are responsible for the opinions expressed by the correspondents. Authors will be invited to reply promptly.

## Comment on "Robust Nonlinear Control of a Hypersonic Aircraft"

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THE purpose of this comment is to point out a potential error in the form of the aerodynamic lift coefficient given in Appendix A of Ref. 1 and its impact on the analysis. The expression for lift coefficient is given in the paper [Eq. (A8)] as

$$C_L = v_9 \alpha (0.493 + v_{10} 1.91/M)$$

The hypersonic vehicle appears to incorporate an elevator as can be observed in the expression for pitching moment [Eq. (A13)]. However, the lift coefficient seems to be unaffected by the elevator deflection  $\delta E$ . Such a flight vehicle would have a highly unusual aerodynamic configuration. The effect of the elevator on the lift coefficient is to introduce the so-called nonminimum phase behavior in the pitch-plane response of tail-controlled flight vehicles. The responses in Fig. 3 of Ref. 1 do not exhibit any such behavior. As is well known in flight dynamics, the nonminimum phase zero introduces a performance bound on the flight control system.

This unusual aerodynamic model explains why the authors get such clean forms for the transformed models given in Eqs. (54) and (55). In real flight vehicles, expressions (54) and (55) can only be obtained approximately, by assuming that the effect of the elevator deflection on the lift coefficient is small. No such assumption is stated in Ref. 1. Note that the stability analysis of approximate feedback linearized flight control systems are much more complex than exactly transformable systems.

Finally, the presence of the nonminimum phase zero in the pitch-plane dynamics is a source of concern to the designers because it is one of the leading causes of nonrobust flight control system behavior. The analysis given in the paper will be much more valuable if this crucial aspect of the flight control problem is included.

## Reference

<sup>1</sup>Wang, Q., and Stengel, R. F., "Robust Nonlinear Control of a Hypersonic Aircraft," *Journal of Guidance, Control, and Dynamics*, Vol. 23, No. 4, 2000, pp. 577–585.

## Reply by the Authors to P. K. Menon

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REFERENCE 1 presents a method for designing robust flight control systems based on a nonlinear dynamic model of the aircraft. The model chosen to demonstrate the new technique is a simplification of the hypersonic aircraft model presented in Ref. 2. The mathematical description of the notional winged-cone aircraft presented in the reference is relatively detailed, and it includes the lifting effect as well as the pitching-moment effect of elevator control deflection. The lifting effect is not included in the simplified model used for design and analysis in our paper.<sup>1</sup>

The nonminimum-phase-zero effect of pitch control using an aft control surface is well known,3 and its impact on nonlinear-dynamicinverse control such as that used in our paper1 is thoroughly discussed in Refs. 4-6. The problem is best understood by considering the equivalent linear model. If the output vector includes normal load factor or another variable dependent on it, such as the flight-path angle, the aft control surface's lifting effect causes the corresponding transfer function to have one or more positive zeros, that is, in the right half of the s plane. Any control law that attempts to invert the transfer function literally is itself unstable because it contains a pole in the right half of the s plane. If the lifting effect is small in comparison to the pitching effect, a satisfactory inverting controllaw can be designed by simply ignoring the effect in the design process, as in Refs. 4 and 6. Another alternative is to command a related variable, for example, pitch angle rather than flight-path angle, whose linear transfer function does not possess a lift-induced right-half-plane zero. A third possibility is to develop an optimal controller that is, in some sense, equivalent to the inverse controller.5

If the elevator lifting effect is large, the controller designed without regard to the effect should be tested against a simulation that includes the effect to assure that the controller is indeed satisfactory. Reference 1 is not intended to address the nonminimum-phase-zero issue, and the controller is not tested against a simulation containing that effect. The principal goals of that paper are as follows.

- 1) Present a method for designing nonlinear control laws that are robust against a large number of system uncertainties and that minimize a design cost function that trades off many stability and performance criteria.
- 2) Compare the nonlinear robust control law with an earlier linear robust control law that was designed for the same hypersonic air craft model and robustness goals.  $^7$

The design approach presented in Ref. 1 is readily modified to include not only the elevator lifting effect but a wide range of nonlinear details of an aircraft dynamic model. To summarize the approach, it uses Monte Carlo simulation of the aircraft dynamic model to

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